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Source: *Biometrika*, Vol. 32, No. 3/4 (Apr., 1942), pp. 243-262
Published by: Biometrika Trust
Stable URL: http://www.jstor.org/stable/2332128

THE TRANSFORMATION OF DATA FROM ENTOMOLOGICAL FIELD EXPERIMENTS SO THAT THE ANALYSIS OF VARIANCE BECOMES APPLICABLE†

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1. Introductory

The present paper deals with experiments on the control of insects in the field. In such experimental work the problem to be investigated is whether more insects survive on plots which have been subjected to one treatment than on plots subjected to another. It will be shown in the present paper that the numbers of insects found per plot must vary in such a way that one cannot, strictly, subject the results to the analysis of variance, and it is proposed to find how the data may be transformed so that analysis of variance becomes applicable. Such transformation has been discussed by Bartlett (1936*a*, *b*) in connexion with entomological experiments, and by Tippett (1934) in connexion with industrial experiments.

2. Experimental results considered

The data used in the following work are results from seven insecticidal experiments arranged by the author at Chatham, Ontario. The work was carried out with replicated blocks containing plots subjected to treatments of which the assignment was random. This procedure, normal in agronomic work, was supplemented by one repetition of each treatment within a block. The assignment of the repetition of a treatment was independent of the first for that treatment, except that, of course, the same plot could not be chosen twice. This repetition was carried out to obtain estimates of variability within blocks. In these experiments complete counts were not made but random sampling was employed. Experiments on *Pyrausta nubilalis* Hubn., reported by Beall *et al.* (1939), for which results are shown in Tables 1 and 2, were made on one area at two different periods, whereas experiments on *Leptinotarsa decemlineata* Say, for which results are indicated in Tables 3 and 4, were carried out on contiguous areas at the same time. Three similar experiments were carried out in one place on the tobacco hornworm, *Phlegethontius quinquemaculata* Haw., for which the data are shown in Tables 5–7. Reference is also made to the data from a uniformity trial on insects of Beall (1939).

† Publication No. 2101, Division of Entomology, Science Service, Department of Agriculture, Ottawa, Canada.

Table 1. *Numbers of an insect,* Pyrausta nubilalis, *per plot. Experiment I*

Treatment	Block 1	2	3	4	5	6	7	8	9	10
1	15	23	21	31	22	14	18	11	21	34
1	27	20	23	33	34	27	17	13	20	26
2	19	12	34	16	20	10	24	23	14	13
2	11	28	37	16	26	18	19	13	10	9
3	16	15	22	25	13	21	18	38	27	10
3	19	16	18	21	19	24	21	18	12	18
4	14	23	10	19	17	18	7	18	8	17
4	34	21	9	34	19	9	15	16	12	12
5	16	16	19	26	15	11	18	23	27	6
5	23	12	12	10	12	17	13	21	12	9
6	12	14	17	10	14	24	24	17	7	3
6	15	16	15	28	13	22	11	7	5	4
7	43	28	35	36	50	69	62	63	42	40
7	47	81	30	69	35	29	71	47	50	43

Table 2. *Numbers of an insect,* Pyrausta nubilalis, *per plot. Experiment II*

Treatment	Block 1	2	3	4	5	6	7	8	9	10
1	32	38	27	7	13	14	26	25	22	30
1	18	40	39	12	19	26	30	19	18	28
2	6	23	8	4	3	18	26	27	17	19
2	9	14	20	13	15	14	15	19	19	10
3	10	21	25	10	13	20	33	48	28	27
3	4	21	26	4	9	14	30	18	27	18
4	2	17	11	3	10	10	26	13	22	17
4	24	13	13	10	6	14	28	11	34	7
5	13	2	5	0	18	10	33	23	20	34
5	17	22	23	8	14	16	26	22	15	34
6	13	10	21	4	10	8	17	15	13	16
6	17	9	29	5	18	15	19	16	27	23
7	37	58	28	11	24	44	30	44	56	45
7	44	71	55	20	26	27	43	52	39	58

Table 3. *Numbers of an insect,* Leptinotarsa decemlineata, *per plot. Experiment III*

Treatment	Block						
	1	2	3	4	5	6	7
1	305	391	420	355	287	175	454
1	207	364	639	527	293	248	397
2	97	49	21	12	3	10	10
2	93	51	25	37	4	12	1
3	270	105	341	469	82	57	221
3	153	190	348	212	100	285	309
4	7	42	34	8	1	10	4
4	12	2	22	4	1	3	3

Table 4. *Numbers of an insect,* Leptinotarsa decemlineata, *per plot. Experiment IV*

Treatment	Block					
	1	2	3	4	5	6
1	253	145	309	665	99	93
1	239	265	166	230	302	237
2	16	13	74	110	14	5
2	95	54	159	108	14	13
3	18	130	165	137	153	78
3	40	137	118	142	239	63
4	2	0	22	6	129	3
4	2	1	31	8	9	8

Table 5. *Numbers of an insect,* Phlegethontius quinquemaculata, *per plot. Experiment V*

Treatment	Block					
	1	2	3	4	5	6
1	6	5	6	13	6	11
1	4	15	13	6	10	15
2	0	1	1	0	1	1
2	2	2	1	4	1	1
3	15	17	22	28	8	16
3	12	22	16	11	13	25

Table 6. *Numbers of an insect,* Phlegethontius quinquemaculata, *per plot. Experiment VI*

Treatment	Block 1	2	3	4	5	6
1	12	13	9	4	11	4
1	13	9	5	7	5	10
2	13	6	8	1	5	7
2	20	9	9	4	12	7
3	7	9	5	4	8	9
3	7	9	4	7	3	2
4	1	1	0	1	4	3
4	2	2	2	1	4	5
5	13	7	12	3	6	11
5	11	5	4	1	9	8
6	7	6	8	9	6	5
6	8	10	2	4	4	12

Table 7. *Numbers of an insect,* Phlegethontius quinquemaculata, *per plot. Experiment VII*

Treatment	Block 1	2	3	4	5	6
1	10	20	14	10	17	14
1	7	14	12	23	20	13
2	11	21	16	17	19	7
2	17	11	14	17	21	13
3	0	7	3	2	3	1
3	1	2	1	1	0	4
4	3	12	4	5	5	2
4	5	6	3	5	5	4
5	3	3	3	1	3	6
5	5	5	6	1	2	4
6	11	15	15	13	26	24
6	9	22	16	10	26	13

3. The relationship between the standard deviation and the mean in the experimental data

If x is the number of insects on one of a group of small contiguous areas, say plots, within a larger area, say a block, let the expectation of x over all these plots be M and the standard deviation be σ; then over a number of the larger areas, when the insects are distributed in a completely random fashion, from the Poisson distribution,

$$\sigma^2 = M. \tag{1}$$

As is discussed by 'Student' (1919) one cannot, however, anticipate that (1) will be satisfied when organisms occur in groups, as, say, when insects come from masses of eggs, or when there is a change in expectation from plot to plot within a block. Generally, σ^2 will tend to be greater than M and we can only say

$$\sigma^2 = f(M). \tag{2}$$

The form of $f(M)$, in (2), must be considered carefully, since it bears on the form of the transformation which may be developed to make the standard deviation independent of the mean.

In dealing with (2), Bartlett (1936*a*) started by supposing that, approximately,

$$\sigma^2 = KM, \tag{3}$$

where K is a constant. Generally, in field data, however, the relationship between σ^2 and M, or of their respective estimates, s^2 and $\bar{x}$, does not, as in Fig. 1, appear to be linear; rather, the departure of s^2 from $\bar{x}$ becomes disproportionately great as $\bar{x}$ increases. This relationship between departures and the magnitude of the mean has been discussed by Clapham (1936) in connexion with data on the distribution of organisms differing from insects as much as flowering plants, and he showed that only those distributions with very low mean have the squared standard deviation close to the mean.

Our discussion above on the shortcomings of (3) suggests the conclusion that

$$\sigma^2 - M \propto M \tag{4}$$

is generally untrue. We propose to consider the possibility that the curvilinearity of (2) might be better met by supposing that

$$\sigma^2 - M \propto M^2. \tag{5}$$

Equation (5) leads to

$$\sigma^2 = M + kM^2, \tag{6}$$

where k is a constant. It will be noticed that

$$k = (\sigma^2 - M)\, M^{-2} \tag{7}$$

is the Charlier coefficient of disturbance from a Poisson distribution. This coefficient was employed by Beall (1935).

It is possible to consider the suitability of (3), as compared with (6), by finding how, respectively, they fit observations on s^2 and $\bar{x}$. To fit exactly is difficult, and it was found necessary to fall back on an empirical determination of K and of k; thus, if there are a number of pairs of estimates, $\bar{x}$ and s^2, from (3) and (6) we estimate

$$K = \Sigma s^2 / \Sigma \bar{x}, \tag{8}$$

$$k = (\Sigma s^2 - \Sigma \bar{x}) / \Sigma \bar{x}^2, \tag{9}$$

where Σ represents the summation over all pairs.

Since in the work presented in § 2, $\bar{x}$ and s^2, being based on only two observations, are highly variable, these experiments do not show clearly the suitability of (3) and (6). Accordingly, reference is made instead to the data from the uniformity trial on *Leptinotarsa decemlineata* Say of Beall (1939). When the mean and standard deviation of 144 sampling units within each of 16 areas were considered, the estimates from (8) and (9) were $K = 2{\cdot}405$ and $k = 0{\cdot}2548$. For these values from (1), (3) and (6), curves, described as lines 1, 2 and 3 respectively, are plotted in Fig. 1, the observed values of mean and squared standard deviation

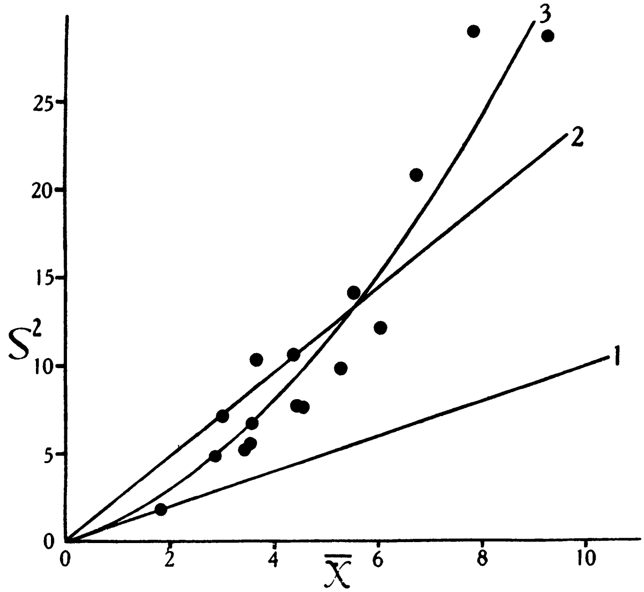

Fig. 1. The squared standard deviation plotted against the mean for 144 small areas within each of 16 large areas; line 1 is from equation (1), line 2 from (3) and line 3 from (6). The counts had been made on *Leptinotarsa decemlineata* Say.

are also shown. In the cases where the mean is near unity the departure of the squared standard deviation from the mean, i.e. from line 1, appears to be trivial, but as the mean increases the departure becomes more marked. It can be seen that the observations lie more snugly about line 3 from (6) than about line 2 from (3). Generally, for the data from field studies the same effect has been observed. Such results suggest that (6) may be generally a better approximation to the form of $f(M)$ than (3) and make it preferable to proceed with the analysis of data from the assumption (6).

4. The transformations of field data

Fig. 1 shows clearly how, within an area, the variability of the numbers of insects on sub-areas is related to the mean number of insects per sub-area. This relationship will make invalid the use of the analysis of variance on experimental results involving counts on insects, since the expectation of the variance should be the same for all plots. To overcome this invalidity, Bartlett (1936*a*) suggested transforming the observations, x, from the basis of (3). The transformation found was $x^{\frac{1}{2}}$, which Bartlett modified to $(x+\frac{1}{2})^{\frac{1}{2}}$. From § 3 it was seen, however, that for field data the relationship between standard deviation and mean may be represented better by equation (6) than by (3), and, since the form of the transformation depends on the form of $f(M)$, a fresh transformation must be sought. A transformation, as is developed in the Appendix to the present paper, is suggested by the method of Tippett (1934), i.e.

$$x' = k^{-\frac{1}{2}} \sinh^{-1} (kx)^{\frac{1}{2}}. \tag{10}$$

An advance note of this transformation was published by Beall (1940). The adequacy of this transformation must be judged from the extent to which it stabilizes variability. In (10), if we express $\sinh^{-1}(kx)^{\frac{1}{2}}$, when $kx < 1$, as a well-known series, we have

$$x' = x^{\frac{1}{2}} - \tfrac{1}{6} kx^{\frac{3}{2}} + \tfrac{3}{40} k^2 x^{\frac{5}{2}} - \tfrac{5}{112} k^3 x^{\frac{7}{2}} + \dots, \tag{11}$$

where it is obvious that for $k = 0$, $x' = x^{\frac{1}{2}}$. Of course, for large values of kx, x' varies almost as $\log x$, or as the $\log(x+1)$ used by Williams (1937), and so our proposed expansion may be regarded, for practical purposes, as embracing the root and logarithmic transformations.

Table 8 gives the transformation, (10), for a probable range of observations, x, and for k at intervals which will probably be close enough for practical purposes. This table was computed in part by inverse interpolation from the table of hyperbolic functions of the *Smithsonian Mathematical Tables* (Becker & Van Orstrand, 1931), and in part from (12). Should values of x' be required outside those of Table 8, these can conveniently be calculated from

$$x' = k^{-\frac{1}{2}} \log_e \{(kx)^{\frac{1}{2}} + (1+kx)^{\frac{1}{2}}\}. \tag{12}$$

In preparing Table 8 the question arose of whether, instead of dealing with $k^{-\frac{1}{2}} \sinh^{-1}(kx)^{\frac{1}{2}}$, one should not use $k^{-\frac{1}{2}} \sinh^{-1}\{k^{\frac{1}{2}}(x+\frac{1}{2})^{\frac{1}{2}}\}$ in the same way as Bartlett (1936*a*) dealt with the transformation, $(x+\frac{1}{2})^{\frac{1}{2}}$, instead of $x^{\frac{1}{2}}$. This modification was rejected on the basis of results of the transformation, as discussed in § 5, since it was found that the addition of $\frac{1}{2}$ made little difference and did not give, consistently, an improvement.

For field data, in making the transformation (10), it is necessary to estimate the value of k empirically by (9) for which estimates $\bar{x}$, of the mean and s, of the standard deviation, must be found. The most obvious method in practice of making these estimates seems to be to put more than one plot subjected to a given treatment in a block and so to estimate the chance variation of results for a plot within a block. In the present work, as is discussed in § 2, two plots were subjected to a given treatment in each block and this is probably good practice.

Table 8. *The transformation* $x' = k^{-\frac{1}{2}} \sinh^{-1} (kx)^{\frac{1}{2}}$

x \ k	0·00	0·02	0·04	0·06	0·08	0·10	0·15	0·20	0·25	0·30	0·40	0·50	0·60	0·80	1·00
0	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00
1	1·00	1·00	0·99	0·99	0·99	0·98	0·98	0·97	0·96	0·96	0·94	0·93	0·92	0·90	0·88
2	1·41	1·40	1·40	1·39	1·38	1·37	1·35	1·33	1·32	1·30	1·27	1·25	1·22	1·18	1·15
3	1·73	1·72	1·70	1·68	1·67	1·66	1·62	1·59	1·57	1·54	1·50	1·46	1·42	1·37	1·32
4	2·00	1·97	1·95	1·93	1·91	1·89	1·84	1·80	1·76	1·73	1·67	1·62	1·58	1·50	1·44
5	2·24	2·20	2·17	2·14	2·11	2·08	2·02	1·97	1·92	1·88	1·81	1·75	1·70	1·61	1·54
6	2·45	2·40	2·36	2·32	2·29	2·25	2·18	2·12	2·06	2·01	1·93	1·86	1·80	1·71	1·63
7	2·65	2·59	2·54	2·49	2·45	2·41	2·32	2·25	2·18	2·13	2·04	1·96	1·89	1·78	1·70
8	2·83	2·76	2·70	2·64	2·59	2·54	2·45	2·36	2·29	2·23	2·13	2·04	1·97	1·85	1·76
9	3·00	2·92	2·84	2·78	2·72	2·67	2·56	2·47	2·39	2·32	2·21	2·12	2·04	1·92	1·82
10	3·16	3·07	2·98	2·91	2·85	2·79	2·66	2·56	2·48	2·40	2·28	2·18	2·10	1·97	1·87
11	3·32	3·21	3·11	3·03	2·96	2·89	2·76	2·65	2·56	2·48	2·35	2·25	2·16	2·02	1·91
12	3·46	3·34	3·23	3·14	3·07	3·00	2·85	2·73	2·63	2·55	2·41	2·30	2·21	2·07	1·96
13	3·61	3·47	3·35	3·25	3·17	3·09	2·93	2·81	2·70	2·62	2·47	2·36	2·26	2·11	1·99
14	3·74	3·59	3·46	3·35	3·26	3·18	3·01	2·88	2·77	2·68	2·52	2·40	2·31	2·15	2·03
15	3·87	3·70	3·56	3·45	3·35	3·26	3·08	2·94	2·83	2·73	2·57	2·45	2·35	2·19	2·06
16	4·00	3·81	3·66	3·54	3·43	3·34	3·15	3·01	2·89	2·79	2·62	2·49	2·39	2·22	2·09
17	4·12	3·92	3·76	3·63	3·51	3·42	3·22	3·07	2·94	2·84	2·67	2·53	2·42	2·25	2·12
18	4·24	4·02	3·85	3·71	3·59	3·49	3·28	3·12	2·99	2·88	2·71	2·57	2·46	2·28	2·15
19	4·36	4·12	3·94	3·79	3·67	3·56	3·34	3·18	3·04	2·93	2·75	2·61	2·49	2·31	2·18
20	4·47	4·22	4·02	3·87	3·74	3·62	3·40	3·23	3·09	2·97	2·79	2·64	2·52	2·34	2·20
21	4·58	4·31	4·11	3·94	3·80	3·69	3·46	3·28	3·13	3·01	2·82	2·68	2·56	2·37	2·23
22	4·69	4·40	4·18	4·01	3·87	3·75	3·51	3·32	3·18	3·05	2·86	2·71	2·58	2·39	2·25
23	4·80	4·49	4·26	4·08	3·93	3·81	3·56	3·37	3·22	3·09	2·89	2·74	2·61	2·42	2·27
24	4·90	4·57	4·34	4·15	3·99	3·86	3·61	3·41	3·26	3·13	2·92	2·77	2·64	2·44	2·29
25	5·00	4·66	4·41	4·21	4·05	3·92	3·65	3·45	3·29	3·16	2·95	2·79	2·66	2·46	2·31
26	5·10	4·74	4·48	4·27	4·11	3·97	3·70	3·49	3·33	3·20	2·98	2·82	2·69	2·48	2·33
27	5·20	4·82	4·54	4·33	4·16	4·02	3·74	3·53	3·37	3·23	3·01	2·85	2·71	2·51	2·35
28	5·29	4·89	4·61	4·39	4·22	4·07	3·78	3·57	3·40	3·26	3·04	2·87	2·73	2·53	2·37
29	5·39	4·97	4·67	4·45	4·27	4·12	3·82	3·61	3·43	3·29	3·07	2·89	2·76	2·54	2·39
30	5·48	5·04	4·74	4·51	4·32	4·16	3·86	3·64	3·46	3·32	3·09	2·92	2·78	2·56	2·40
31	5·57	5·11	4·80	4·56	4·37	4·21	3·90	3·67	3·50	3·35	3·12	2·94	2·80	2·58	2·42
32	5·66	5·18	4·86	4·61	4·42	4·25	3·94	3·71	3·53	3·38	3·14	2·96	2·82	2·60	2·43
33	5·74	5·25	4·91	4·66	4·46	4·30	3·98	3·74	3·55	3·40	3·16	2·98	2·84	2·62	2·45
34	5·83	5·32	4·97	4·71	4·51	4·34	4·01	3·77	3·58	3·43	3·19	3·00	2·86	2·63	2·46
35	5·92	5·38	5·03	4·76	4·55	4·38	4·05	3·80	3·61	3·45	3·21	3·02	2·88	2·65	2·48
36	6·00	5·45	5·08	4·81	4·59	4·42	4·08	3·83	3·64	3·48	3·23	3·04	2·89	2·66	2·49
37	6·08	5·51	5·13	4·85	4·64	4·46	4·11	3·86	3·66	3·50	3·25	3·06	2·91	2·68	2·51
38	6·16	5·57	5·18	4·90	4·68	4·49	4·14	3·89	3·69	3·53	3·27	3·08	2·93	2·69	2·52
39	6·24	5·63	5·23	4·94	4·72	4·53	4·17	3·91	3·71	3·55	3·29	3·10	2·94	2·71	2·53

40	6·32	5·69	5·28	4·99	4·76	4·57	4·20	3·94	3·74	3·57	3·31	3·12	2·96	2·72	2·54
41	6·40	5·75	5·33	5·03	4·79	4·60	4·23	3·97	3·76	3·59	3·33	3·13	2·98	2·73	2·56
42	6·48	5·81	5·38	5·07	4·83	4·63	4·26	3·99	3·78	3·61	3·35	3·15	2·99	2·75	2·57
43	6·56	5·86	5·43	5·11	4·87	4·67	4·29	4·02	3·81	3·63	3·37	3·17	3·01	2·76	2·58
44	6·63	5·92	5·47	5·15	4·90	4·70	4·32	4·04	3·83	3·65	3·39	3·18	3·02	2·77	2·59
45	6·71	5·97	5·52	5·19	4·94	4·73	4·35	4·07	3·85	3·67	3·40	3·20	3·03	2·79	2·60
46	6·78	6·03	5·56	5·23	4·97	4·76	4·37	4·09	3·87	3·69	3·42	3·21	3·05	2·80	2·61
47	6·86	6·08	5·61	5·27	5·01	4·80	4·40	4·11	3·89	3·71	3·44	3·23	3·06	2·81	2·62
48	6·93	6·13	5·65	5·30	5·04	4·83	4·42	4·13	3·91	3·73	3·45	3·24	3·08	2·82	2·63
49	7·00	6·18	5·69	5·34	5·07	4·85	4·45	4·16	3·93	3·75	3·47	3·26	3·09	2·83	2·64
50	7·07	6·23	5·73	5·38	5·10	4·88	4·47	4·18	3·95	3·77	3·48	3·27	3·10	2·84	2·65
55	7·42	6·47	5·93	5·55	5·26	5·02	4·59	4·28	4·04	3·85	3·56	3·34	3·16	2·90	2·70
60	7·75	6·70	6·11	5·70	5·39	5·15	4·70	4·37	4·13	3·93	3·62	3·40	3·22	2·94	2·74
65	8·06	6·91	6·28	5·85	5·52	5·27	4·79	4·46	4·20	4·00	3·69	3·45	3·27	2·99	2·78
70	8·37	7·11	6·44	5·98	5·64	5·38	4·88	4·54	4·28	4·07	3·74	3·50	3·32	3·03	2·82
75	8·66	7·30	6·58	6·11	5·76	5·48	4·97	4·61	4·34	4·13	3·80	3·55	3·36	3·07	2·86
80	8·94	7·47	6·73	6·23	5·86	5·57	5·05	4·68	4·41	4·19	3·85	3·60	3·40	3·10	2·89
85	9·22	7·64	6·86	6·34	5·96	5·66	5·13	4·75	4·47	4·24	3·90	3·64	3·44	3·14	2·92
90	9·49	7·80	6·98	6·45	6·06	5·75	5·20	4·81	4·52	4·29	3·94	3·68	3·48	3·17	2·95
95	9·75	7·96	7·10	6·55	6·15	5·83	5·26	4·87	4·57	4·34	3·98	3·72	3·51	3·20	2·97
100	10·00	8·10	7·22	6·65	6·23	5·91	5·33	4·93	4·62	4·39	4·02	3·75	3·54	3·23	3·00
110	10·49	8·38	7·43	6·83	6·39	6·05	5·45	5·03	4·72	4·47	4·10	3·82	3·60	3·28	3·05
120	10·95	8·64	7·63	6·99	6·54	6·18	5·56	5·13	4·80	4·55	4·16	3·88	3·66	3·33	3·09
130	11·40	8·88	7·81	7·15	6·67	6·31	5·66	5·21	4·88	4·62	4·23	3·94	3·71	3·37	3·13
140	11·83	9·10	7·98	7·29	6·80	6·42	5·75	5·30	4·96	4·69	4·29	3·99	3·76	3·42	3·17
150	12·25	9·31	8·14	7·42	6·91	6·53	5·84	5·37	5·02	4·75	4·34	4·04	3·80	3·45	3·20
160	12·65	9·51	8·29	7·55	7·02	6·62	5·92	5·44	5·09	4·81	4·39	4·08	3·84	3·49	3·23
170	13·04	9·70	8·43	7·67	7·13	6·72	6·00	5·51	5·15	4·86	4·44	4·13	3·88	3·52	3·26
180	13·42	9·88	8·57	7·78	7·23	6·81	6·07	5·57	5·20	4·92	4·48	4·17	3·92	3·56	3·29
190	13·78	10·05	8·69	7·88	7·32	6·89	6·14	5·63	5·26	4·96	4·52	4·20	3·95	3·59	3·32
200	14·14	10·21	8·81	7·98	7·41	6·97	6·20	5·69	5·31	5·01	4·57	4·24	3·99	3·61	3·34
210	14·49	10·36	8·93	8·08	7·49	7·04	6·26	5·74	5·36	5·05	4·60	4·27	4·02	3·64	3·37
220	14·83	10·51	9·04	8·17	7·57	7·11	6·32	5·79	5·40	5·10	4·64	4·31	4·05	3·67	3·39
230	15·17	10·65	9·14	8·26	7·65	7·18	6·38	5·84	5·45	5·14	4·68	4·34	4·08	3·69	3·41
240	15·49	10·79	9·25	8·34	7·72	7·25	6·43	5·89	5·49	5·18	4·71	4·37	4·11	3·72	3·43
250	15·81	10·92	9·34	8·42	7·79	7·31	6·49	5·93	5·53	5·21	4·74	4·40	4·13	3·74	3·45
260	16·12	11·05	9·44	8·50	7·86	7·37	6·54	5·98	5·57	5·25	4·77	4·42	4·16	3·76	3·47
270	16·43	11·17	9·53	8·58	7·92	7·43	6·58	6·02	5·61	5·28	4·80	4·45	4·18	3·78	3·49
280	16·73	11·29	9·61	8·65	7·99	7·49	6·63	6·06	5·64	5·32	4·83	4·48	4·20	3·80	3·51
290	17·03	11·40	9·70	8·72	8·05	7·54	6·68	6·10	5·68	5·35	4·86	4·50	4·23	3·82	3·53
300	17·32	11·51	9·78	8·79	8·10	7·60	6·72	6·14	5·71	5·38	4·88	4·53	4·25	3·84	3·55

In the special case where there are two plots for the ith treatment ($i = 1, \ldots, n$) in the jth block ($j = 1, \ldots, N$), and so two observations, x_{ij1} and x_{ij2}, the estimate of the mean will be written $x_{ij.}$ and of the squared standard deviation

$$s_{ij}^2 = \tfrac{1}{2}(x_{ij1} - x_{ij2})^2. \tag{13}$$

Then from (9), we estimate k from

$$k = 2\left\{\sum_{i=1}^{n}\sum_{j=1}^{N}(x_{ij1} - x_{ij2})^2 - \sum_{i=1}^{n}\sum_{j=1}^{N}(x_{ij1} + x_{ij2})\right\}\left\{\sum_{i=1}^{n}\sum_{j=1}^{N}(x_{ij1} + x_{ij2})^2\right\}^{-1} \tag{14}$$

and the calculation is very light.

5. Results showing the effect of the transformation on the variability of data

The adequacy of our proposed transformation may be judged in two ways: first, with respect to its effect, which we shall consider in the present section, on the differences between repetitions of a treatment within a block, and secondly, with respect to its effect, which we shall consider in § 6, on the behaviour of the quantities submitted to the analysis of variance.

It is a fundamental assumption in the analysis of variance that the chance variability for each plot shall be, when the effect of block and of treatment are removed, normally distributed with a standard deviation common to all plots, in which situation of course the standard deviation of the chance variability for a given plot is independent of the expectation for that plot. In the data of the present work, where each treatment is repeated in each block, it is possible to examine the estimates of this standard deviation, s_{ij}, and of the expectation, $x_{ij.}$. For a clear graphical illustration of the situation consider Fig. 2, as obtained from the original data of Experiment III on *Leptinotarsa decemlineata*, where s_{ij} is plotted against $x_{ij.}$, and contrast this situation with that obtaining for the corresponding quantities s'_{ij} and $x'_{ij.}$, obtained after transformation ($k = 0{\cdot}08$) in Fig. 3.

In Fig. 2 the points are widely scattered as is natural from a sample of two; nevertheless, it is apparent that for the smallest values of $x_{ij.}$ the values of s_{ij} are correspondingly small and fall in a close group. In Fig. 3 the cluster of observations in the lower left-hand corner of the previous diagram has disappeared, and generally the scatter appears to be independent of $x'_{ij.}$, so that apparently the transformation gave satisfactory results. The nature of the material involved is such that it does not seem possible to examine the relationship under consideration more exactly, nor to summarize exactly the corresponding results for the other treatments; it can only be said that the same type of result appeared although the magnitude of the relationship before transformation depended on the magnitude of the differences between the effects of treatments.

The results shown in Figs. 2 and 3 suggest that the proposed transformation has tended to make the standard deviation independent of the mean, in accordance with the assumptions underlying the analysis of variance. In using this procedure one actually assumes, more broadly, that a common standard deviation exists, so that the homoscedasticity of observations before and after transformation should be tested. Thus it is assumed that x_{ij1} and x_{ij2} are observations from a normal

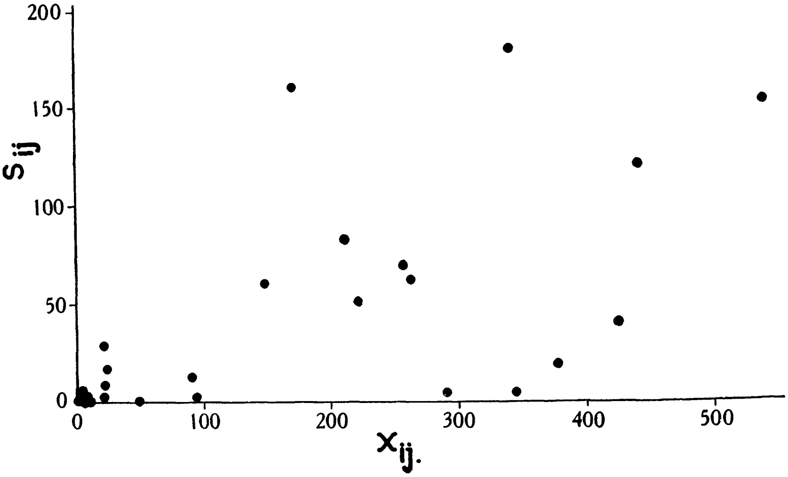

Fig. 2. The standard deviation and mean as estimated from plots by pairs, with untransformed data on *Leptinotarsa decemlineata*.

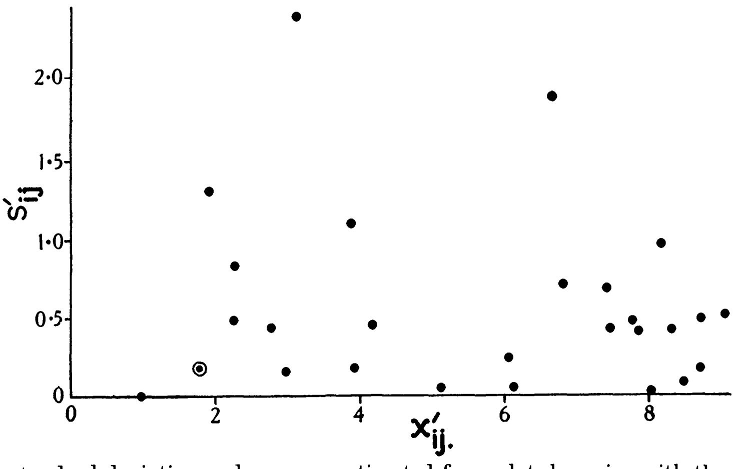

Fig. 3. The standard deviation and mean as estimated from plots by pairs, with the transformed data on *Leptinotarsa decemlineata* Say, i.e. using $x' = k^{-\frac{1}{2}} \sinh^{-1} (kx)^{\frac{1}{2}}$, $(k = 0{\cdot}08)$.

population with a standard deviation, σ, which is independent of i and j. Then $\left\{\sum_{k=1}^{2} (x_{ijk} - x_{ij.})^2\right\} \frac{1}{\sigma^2}$ is distributed as χ^2 with one degree of freedom.† Accordingly,

† In the L_1 test, discussed by Nayer (1936), this case of estimates of standard deviation with one degree of freedom is troublesome since zero values tend to arise when dealing with grouped or integral observations. When this is the case L_1, which is the ratio of an arithmetic to a geometric mean of sums of squares, cannot be calculated. The present treatment may therefore have a wider application.

$y_{ij} = (x_{ij1} - x_{ij2})/\sqrt{2}\sigma$ should be distributed normally with unit standard deviation for all i and j. In order to test the hypothesis of normality with unit standard deviation it is only necessary to test for leptokurtosis; for the distribution must be symmetrical since the sign of differences, and therefore of y_{ij}, is a matter of chance. Since the number of items involved will almost certainly be < 100, and since the population mean is zero, the w_n criterion of Geary (1935) will provide an appropriate test. In using this criterion we must find the ratio of the mean deviation to the standard deviation, i.e.

$$w_n = \left\{\sum_{i=1}^{n}\sum_{j=1}^{N} | x_{ij1} - x_{ij2} |\right\}\left\{nN \sum_{i=1}^{n}\sum_{j=1}^{N}(x_{ij1} - x_{ij2})^2\right\}^{-\frac{1}{2}} \tag{15}$$

Of course, values of w_n may be calculated for transformed data by substitution of x'_{ijk} for x_{ijk}.

Table 9. *The w_n test on the homoscedasticity of counts by plots within a block for six field experiments*

Experiment	nN	Upper 5% limit	Lower 5% limit	Untransformed w_n	Untransformed Departure by S.D.	Transformed (Bartlett) w_n	Transformed (Bartlett) Departure by S.D.	Value of k: Estimated	Value of k: Employed	Transformed (Beall) w_n	Transformed (Beall) Departure by S.D.
I	70	0·841	0·757	0·6659	−5·33	0·7525	−1·91	0·078	0·08	0·7846	−0·64
II	70	0·841	0·757	0·7885	−0·49	0·7807	−0·79	0·046	0·04	0·7499	−2·01
III	28	0·866	0·737	0·5973	−5·33	0·6431	−4·15	0·084	0·08	0·6838	−3·11
IV	24	0·872	0·732	0·5692	−5·67	0·6948	−2·67	0·285	0·30	0·7370	−1·66
V	18	0·881	0·728	0·7554	−1·16	0·8155	+0·13	0·082	0·08	0·7823	−0·58
VI	36	0·857	0·745	0·7959	−0·22	0·8166	+0·38	0·019	0·02	0·8130	+0·28

Values of w_n from the untransformed observations and from the transformed observations, both following Bartlett (i.e. the transformation $(x+\frac{1}{2})^{\frac{1}{2}}$) and following the line suggested in the present paper, are shown in Table 9 for the field data of Tables 1–6. For the second transformation the values of k as calculated from (14) are shown as well as the nearest value of k entered in Table 8. For each experiment the value of nN and also the 0·05 limits of probability, from Geary (1935), are shown. There are also shown the departures of observed w_n from the expected value in terms of the standard deviation, a useful criterion since the distribution of w_n is almost normal. From Table 9 it can be seen that out of the three experiments in which w_n fell beyond the lower 5% limit of probability for the untransformed data and the data transformed as $(x+\frac{1}{2})^{\frac{1}{2}}$, in only one experiment did w_n fall so with the final transformation. The results for Experiment II, in which w_n is decreased by the transformation, are peculiar. Consideration of the departures from the mean in terms of the standard deviation indicates more clearly the improvement effected by each transformation and how the transformation suggested in the present work secures an improvement of the same, but more marked, character than that secured from the transformation of Bartlett. The results suggest that while homoscedasticity may not be attained always, it will be approached by means of the proposed transformation.

6. THE EFFECT OF THE TRANSFORMATION ON THE ANALYSIS OF VARIANCE

As was indicated at the beginning of § 5, our proposed transformation besides making the variability within a block for a repeated treatment the same for all treatments and blocks, should also provide quantities satisfying the assumptions underlying the analysis of variance. Since it is not quite clear how, in so far as the transformation is satisfactory in the first way, it will necessarily be satisfactory in the second, it will be well to consider directly the suitability of our transformed values for the analysis of variance.

In the application of the analysis of variance one would deal with $x_{ij.}$ rather than with x_{ijk} and suppose that

$$x_{ij.} = A + B_i + C_j + D_{ij}, \tag{16}$$

where A is a contribution from the general level of population on the experimental area, B_i the contribution of the ith treatment and C_j the contribution of the jth block. The remainder term, D_{ij}, is called the interaction of treatments and blocks. Of course, the present discussion on the untransformed values, $x_{ij.}$, holds for the transformed values, $x'_{ij.} = \frac{1}{2}(x'_{ij1} + x'_{ij2})$ when the appropriate symbols, A', B'_i, C'_j and D'_{ij} are used.

In material satisfying the conditions underlying the analysis of variance, for the observations under each treatment, the calculated squared standard deviation is

$$s_i^2 = \frac{1}{N-1} \sum_{j=1}^{N} (x_{ij.} - x_{i..})^2. \tag{17}$$

Following the argument of the analysis of variance, $x_{ij.} - x_{i..}$, of which the mean is 0, is an estimate of $C_j + D_{ij}$, in which the two terms are independent; hence the expectation of s_i^2 is

$$\sigma_i^2 = \sigma_{C_j}^2 + \sigma_{D_{ij}}^2, \tag{18}$$

where σ_{Cj} and $\sigma_{D_{ij}}$ are the standard deviations of the parameters, C_j and D_{ij}, respectively, and are independent of treatment. Accordingly, s_i should be independent of treatment and distributed as an estimate of σ_i, having $N-1$ degrees of freedom. Conversely, if $x_{ij.}$ cannot be built of the independent terms of (16), then the various values of s_i will not be distributed as estimates of a single standard deviation. The hypothesis that the values of s_i in any one experiment are estimates of one quantity may be tested.†

† From correspondence with Dr R. W. B. Jackson, the writer has learned that he had arrived independently at the same test.

The results of the tests on the homogeneity of the values, s_i, within the six experiments treated in the present paper are presented in Table 10, where the value of the L_1 criterion is shown for the original data and for the transformed values together with the appropriate 0·05 and 0·01 levels of probability (Nayer, 1936). From Table 10 it can be seen that of the values of L_1 obtained from the original data, all but one are near or beyond the 0·05 level of significance, but that after transformation all are moved in to less significant values. Accordingly, the values of s_i, when calculated from the original data, appear heterogeneous but the corresponding values obtained after the transformation appear homogeneous. Thus it is more probable that the analysis of variance is applicable to the transformed data than to the untransformed.

Table 10. *The homogeneity, as measured by the criterion L_1, of the estimates s_i for various values of i before and after transformation*

	Experiment					
	1	2	3	4	5	6
L_1 before transformation	0·867	0·833	0·325	0·657	0·344	0·680
L_1 after transformation	0·864	0·941	0·766	0·688	0·813	0·730
1 % limit	0·757	0·757	0·604	0·542	0·514	0·583
5 % limit	0·812	0·812	0·707	0·656	0·648	0·673

In Table 10 we have tested the homogeneity of the estimates, s_i, as in § 5 we tested the homogeneity of s_{ij}, that is without reference to the values of the associated means. In view of our original assumptions we are, however, interested in the possibility that the standard deviations, as calculated, might show every sign of being estimates of a common standard deviation and yet be dependent on the associated means. Accordingly, we have investigated such dependence roughly by fitting by least squares a first order regression of s_i on $x_{i..}$. From this fitting we record the sign of the regression as follows:

	Experiment					
	1	2	3	4	5	6
Before transformation	+ *	+ **	+ *	+ **	+	+
After transformation	−	+	−	− *	+	+

By a single asterisk we have indicated cases where the reduction in variability effected by the regression passed the 5 % probability limit and by a double asterisk where it passed the 1 % limit. Several points may be noted. (1) In two cases (Experiments 2 and 6) after transformation the residual sum of squares about the regression was greater than the reduction in squares due to the regression, whereas it was consistently less before transformation. (2) As can be seen above, the regression generally did not effect a significant reduction in variability after transformation but did before (the small number of degrees of freedom made high significance difficult of attainment). (3) After transformation the sign of the regression seemed to be a chance matter, whereas before transformation it was consistently positive. These results suggest that the transformation proposed did tend to make the variability within a given treatment independent of the mean for that treatment.

7. THE EFFECT OF TRANSFORMATION UPON THE CONCLUSIONS FROM THE ANALYSIS OF VARIANCE

It has been shown in §§ 5 and 6 that the analysis of variance can be made on entomological data when a suitable transformation has been effected. It is of practical interest to see what numerical effect such transformation will have upon tests on the significance of, say, the effect of treatment and the significance of differences for treatments.

First, consider the numerical results to be obtained from the analysis of variance (1) without and (2) with transformation. Thus the mean square ascribable to blocks, treatments and their interaction is shown in Table 11, for six experiments of which the data are given in § 2; parallel results are presented for untransformed observations and for observations transformed by (10) with the values of k from Table 9. To facilitate the comparison of the results, the mean square for blocks and for treatments is expressed in terms of the estimate for interaction, as the F of Snedecor (1934), and presented in each case. The transformation of the data has modified the conclusions to be drawn from the analysis of variance in Table 11, in that there are considerable changes in the criterion, F, for treatments or for blocks. In the examples shown the effect of treatments was highly significant in all cases and so the changes introduced by transformation did not alter the conclusions, as would have been the case for less definite effects.

Consider next the effect of transformation on the significance of differences between the means for treatments as tested by the criterion, t, calculated with such estimates of mean square as the interaction of Table 11. For illustration, values of t, from the data on *Leptinotarsa decemlineata* (Experiment III), are shown in Table 12 for each possible comparison of treaments when untransformed data are used, when the transformation, $(x+\frac{1}{2})^{\frac{1}{2}}$, as suggested by Bartlett (1936*a*) is used, and when the transformation, $k^{-\frac{1}{2}}\sinh^{-1}(kx)^{\frac{1}{2}}$, as suggested in the present paper is used. In order that the influence of the level of population under each

Table 11. *The analysis of variance of untransformed and transformed data in six experiments*

Variation	Degrees of freedom	Untransformed data		Transformed data	
		Mean square	F	Mean square	F
Experiment I. *P. nubilalis*					
Between blocks	9	92·8	1·21	0·565	1·66
Between treatments	6	2,839·0	36·95	7·51	22·03
Interaction	54	76·8	—	0·341	—
Experiment II. *P. nubilalis*					
Between blocks	9	577·0	6·72	5·37	10·55
Between treatments	6	1,721·0	20·04	8·69	17·07
Interaction	54	85·9	—	0·509	—
Experiment III. *L. decemlineata*					
Between blocks	6	20,172·0	2·26	4·67	3·03
Between treatments	3	390,932·0	43·77	111·5	72·16
Interaction	18	8,931·0	—	1·54	—
Experiment IV. *L. decemlineata*					
Between blocks	5	12,960·0	2·06	2·06	1·96
Between treatments	3	124,054·0	20·40	20·40	19·41
Interaction	15	6,727·0	—	1·05	—
Experiment V. *P. quinquemaculata*					
Between blocks	5	27·4	2·74	0·349	4·19
Between treatments	2	752·0	75·33	19·5	233·77
Interaction	10	9·98	—	0·083	—
Experiment VI. *P. quinquemaculata*					
Between blocks	5	41·7	4·13	1·50	4·12
Between treatments	5	66·2	6·55	3·33	9·14
Interaction	25	10·1	—	0·365	—

Table 12. *The values of t, in the comparison of means, as calculated from the untransformed and the transformed data of Experiment III on* Leptinotarsa decemlineata

Comparison	Means for untransformed data		t without transformation	t from $(x+\frac{1}{2})^{\frac{1}{2}}$	t from $k^{-\frac{1}{2}}\sinh^{-1}(kx)^{\frac{1}{2}}$
$x_{1..}-x_{2..}$	362	30	+9·27**	+11·33**	+ 9·92**
$x_{1..}-x_{3..}$	362	224	+3·84**	+ 3·52**	+ 2·11*
$x_{1..}-x_{4..}$	362	11	+9·82**	+12·89**	+12·46**
$x_{2..}-x_{3..}$	30	224	−5·43**	− 7·81**	− 7·81**
$x_{2..}-x_{4..}$	30	11	+0·54	+ 1·56	+ 2·54*
$x_{3..}-x_{4..}$	224	11	+5·98**	+ 9·37**	+10·35**

treatment may be judged, there are shown, also in Table 12, the means for the untransformed data. The values of t falling beyond the 0·01 level of significance have been marked with two asterisks and the values beyond the 0·05 level with one. It can be seen that the transformation resulted in a profound alteration in the conclusions. Apparently on account of the dependence of variance on mean in untransformed data, the pooled estimate of variance was originally too low for the treatments which resulted in high populations and too high for the treatments which resulted in low populations. Thus, in the comparison of the first and third treatments, which appeared to have the two highest surviving populations, the value of t calculated from untransformed values was high. In the other extreme case, the comparison between the second and fourth treatments, the value of t, as calculated from untransformed values was very low. It can be seen further, that the first transformation only secured in part the modification in the value of t that was secured by the second transformation.

8. THE PROCEDURE OF TRANSFORMATION IN PRACTICE

The methods which were found applicable in the preceding discussion will now be illustrated in the transformation of the data shown in Table 7 (Experiment VII) on *Phlegethontius quinquemaculata*, of the same type as the experiments previously discussed in the present paper. The steps in the analysis will be set out with the purpose of providing a model for procedure in estimating the constant, k, which will be used to effect a transformation of the data so that the analysis of variance may be made.

Supposing that the experiment has been laid out with a repetition of each treatment in each block, the procedure of estimating k makes it first necessary to find the sum and the absolute difference of each pair of plots subjected to a given treatment in a given plot and then to sum the sums, $\sum_{i=1}^{n}\sum_{j=1}^{N}(x_{ij1}+x_{ij2})$, and the sums squared, $\sum_{i=1}^{n}\sum_{j=1}^{N}(x_{ij1}+x_{ij2})^2$, and also the differences squared, $\sum_{i=1}^{n}\sum_{j=1}^{N}(x_{ij1}-x_{ij2})^2$, over all such pairs and by substituting the results in (14) to find k. In the case being used for an illustration the two plots subjected to the first treatment in each block gave respectively 10 and 7, 20 and 14, 14 and 12, 10 and 23, 17 and 20, 14 and 13, so that

$$\sum_{j=1}^{N}(x_{1j1}+x_{1j2}) = (10+7)+(20+14)+(14+12)+\ldots = 174.$$

Similarly,

$$\sum_{j=1}^{N}(x_{1j1}+x_{1j2})^2 = (10+7)^2+(20+14)^2+\ldots = 5308$$

and similarly,

$$\sum_{j=1}^{N}(x_{1j1}-x_{1j2})^2 = (10-7)^2+(20-14)^2+\ldots = 228.$$

Of course, in estimating k the summations are not limited to one treatment but must be extended over all in the experiments. If this is done we find

$$\sum_{i=1}^{n}\sum_{j=1}^{N}(x_{ij1}+x_{ij2}) = 684, \quad \sum_{i=1}^{n}\sum_{j=1}^{N}(x_{ij1}+x_{ij2})^2 = 19{,}656, \quad \sum_{i=1}^{n}\sum_{j=1}^{N}(x_{ij1}-x_{ij2})^2 = 708.$$

From (14) we estimate $k = \dfrac{2(708-684)}{19{,}656} = 0{\cdot}002,$

and referring to Table 8, p. 250, use $k = 0{\cdot}00$ as the nearest value occurring there. Of course, in this case, the transformation is simply $x^{\frac{1}{2}}$.

Now from the above result it will be possible to replace the observed values of Table 7 with the corresponding transformed values from the first column of Table 8. Thus in Table 7 replace in the first row: 10, 20, 14, 10, 17 and 14, by 3·16, 4·47, 3·74, 3·16, 4·12 and 3·74. With such transformed values we can now proceed to carry out a routine analysis of variance which will be facilitated by working with the sum for each pair of plots in a given block with a given treatment. For example, the final analysis of variance for Experiment VII would be carried out with the values of Table 13.

Table 13. *Transformed and summed values to be used in the analysis of variance for Experiment VII on* P. quinquemaculata

Treatment	Block					
	1	2	3	4	5	6
1	5·81	8·21	7·20	7·96	8·59	7·35
2	7·44	7·90	7·74	8·24	8·94	6·26
3	1·00	4·06	2·73	2·41	1·73	3·00
4	3·97	5·91	3·73	4·48	4·48	3·41
5	3·97	3·97	4·18	2·00	3·14	4·45
6	6·32	8·56	7·87	6·77	10·20	8·51

9. Summary and conclusions

The foregoing work is a study of experimental results from seven field experiments on the control of insects. In such data, the standard deviation of the number of insects per plot varies with the mean. By the transformation, $x' = k^{-\frac{1}{2}}\sinh^{-1}(kx)^{\frac{1}{2}}$, where k is a constant and x an observation, the data were put in a form for which the standard deviation approached a constant independent of the mean. The estimation of the one constant, k, necessary for the transformation was made possible by the design of the experiments with repetition of treatments within blocks. In practice, the transformation gave good results so that

analysis of variance could be made. From the analysis of the transformed data, the results were found to differ markedly from those which would have been obtained from the untransformed data.

ACKNOWLEDGEMENTS

The present work was suggested to the writer by Prof. E. S. Pearson to whom the writer is further indebted for advice as the work progressed. The writer is further beholden to Dr R. W. B. Jackson, to Dr B. L. Welch and to Dr M. S. Bartlett for criticism and advice. Dr G. M. Stirrett and Mr F. A. Stinson very kindly permitted the use of unpublished data from their experiments carried out at Delhi, Ontario, on *Phlegethontius quinquemaculata* Haw.

APPENDIX

As has been said, the transformation of (10) was suggested by the method used by Tippett (1934, p. 61). The procedure is as follows.

It is required to find $x' = f(x)$, such that the standard deviation, $\sigma_{x'}$ of x', shall be approximately constant. Let us write

$$x' = f(M) + f'(M)(x-M) + \dots, \tag{19}$$

where M is the expectation of x and whence, approximately,

$$(x'-M') = f'(M)(x-M), \tag{20}$$

where M' is the expectation of x'. Hence

$$\sigma^2_{x'} = \{f'(M)\}^2 \sigma^2, \tag{21}$$

where σ is the standard deviation of the observations, x. Replacing $\sigma_{x'}$ in (21) by a constant, c, as is the purpose of our operation, and substituting for σ from equation (6), p. 247, we have

$$f'(M) = c(M + kM^2)^{-\frac{1}{2}}, \tag{22}$$

where k is, as has been previously discussed, a constant peculiar to our data. Integrating in (22),

$$f(M) = 2ck^{-\frac{1}{2}} \sinh^{-1} (kM)^{\frac{1}{2}}. \tag{23}$$

From (23) the form of the function suggested is $\sinh^{-1}(kx)^{\frac{1}{2}}$, but it is wise instead to use $k^{-\frac{1}{2}} \sinh^{-1}(kx)^{\frac{1}{2}}$, since the transformation then becomes identical, as shown in (11), with the established transformation, $x^{\frac{1}{2}}$, when $k = 0$.

As Tippett (1934) says: 'This derivation is not mathematically sound, and the result is only justified if on application it is found to be satisfactory.' The writer would have hesitated to have used it had it not already led to useful transformations in cases analogous to the present, namely to $x^{\frac{1}{2}}$ where x comes from a Poisson distribution, to $\sin^{-1} p^{\frac{1}{2}}$ where p comes from a binomial distribution and, according to Tippett, to $\tanh^{-1} r$, where r is the correlation coefficient.

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